

Facts about Improper Integrals	Explanation
What is an Improper Integral? When does an Improper Integral Converge or Diverge?	An improper integral is one in which the interval of integration is unbounded or the integrand has a discontinuity at some point in the interval of integration. If the value of the improper integral is a finite number, then the improper integral converges. Otherwise, it diverges.
Rewriting an Improper Integral using Limits in Order to Compute its Value.	<p>Let a, b be any number in the interval of integration (including the endpoints) and c be a discontinuous point for $f(x)$ within the interval of integration.</p> <p><u>Case 1:</u></p> $\int_{-\infty}^b f(x) dx = \lim_{t \rightarrow -\infty} \int_t^b f(x) dx$ <p><u>Case 2:</u></p> $\int_a^{\infty} f(x) dx = \lim_{t \rightarrow \infty} \int_a^t f(x) dx$ <p><u>Case 3:</u></p> $\int_{-\infty}^{\infty} f(x) dx = \int_{-\infty}^a f(x) dx + \int_a^{\infty} f(x) dx$ <p><u>Case 4:</u></p> $\int_a^c f(x) dx = \lim_{t \rightarrow c^-} \int_a^t f(x) dx$ <p><u>Case 5:</u></p> $\int_c^b f(x) dx = \lim_{t \rightarrow c^+} \int_t^b f(x) dx$ <p><u>Case 6:</u></p> $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$

1. Consider the improper integral $\int_{-\infty}^{\infty} \frac{e^x}{1 + e^{2x}} dx$.

(a) Are there any discontinuities for the function $f(x) = \frac{e^x}{1 + e^{2x}}$ within the range of the limits of integration?

(b) Rewrite the improper integral using limit notation. Note that it may be easier to use $x = 0$ as a reference point.

(c) Evaluate the improper integral.

(d) Does the improper integral converge or diverge?

2. Consider the improper integral $\int_0^{\infty} e^{-x} dx$.

(a) Are there any discontinuities for the function $f(x) = e^{-x}$ within the range of the limits of integration?

(b) Rewrite the improper integral using limit notation.

(c) Evaluate the improper integral.

(d) Does the improper integral converge or diverge?

3. Consider the improper integral $\int_0^1 \frac{1}{\sqrt[3]{x}} dx$.

(a) Are there any discontinuities for the function $f(x) = \frac{1}{\sqrt[3]{x}}$ within the range of the limits of integration?

(b) Rewrite the improper integral using limit notation.

(c) Evaluate the improper integral.

(d) Does the improper integral converge or diverge?

4. What happens when you evaluate $\int_0^{\infty} \frac{1}{x} dx$? How about $\int_1^{\infty} \frac{1}{x} dx$? And $\int_0^1 \frac{1}{x} dx$?